REPUBLIC OF UZBEKISTAN
MINISTRY OF HIGHER EDUCATION, SCIENCE AND INNOVATION

SAMARKAND STATE UNIVERSITY OF
VETERINARY MEDICINE, LIVESTOCK AND BIOTECHNOLOGIES

Teacher of the Department of Information Technologies, Natural and Exact Sciences

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Faculty of Economic

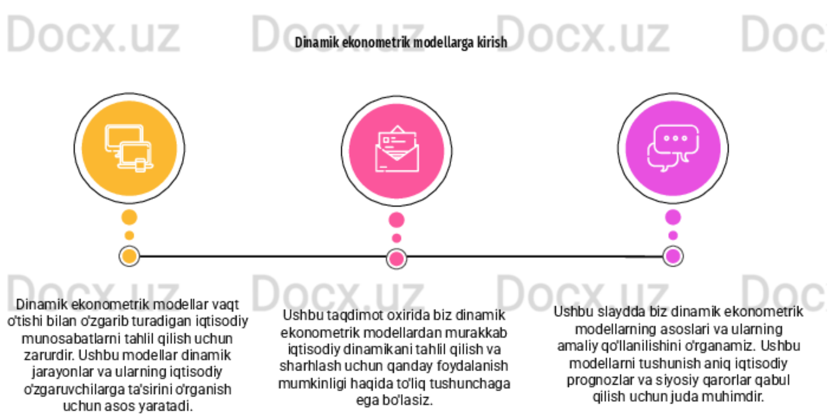
Economic major

For students of group 204 of the 2st year

APPLIED ECONOMETRICS

Dynamic econometric models

PRACTICAL TRAINING WORKSHOP



Samarkand 2026

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APPLIED ECONOMETRICS

Dynamic econometric models

Number of students 15	
Number of sessions	Praktical treaning
Praktical treaning workshops	1. Functional, statistical and correlation relationships 2. Linear regression selection equation 3. Formulas for calculating the correlation coefficient 4. Least squares method in constructing a linear regression equation Perform tasks on the topic
Purpose of the training session	Develop skills on the topic
Pedagogical tasks:	Results of educational activities:
- To provide an understanding of functional, statistical and correlation relationships; - To teach how to construct a linear regression equation; - To verify the least squares method in constructing a linear regression equation using formulas;	- To gain an understanding of functional, statistical and correlation relationships; - To independently study how to construct a linear regression equation; - To independently verify the least squares method in constructing a linear regression equation using formulas in the Excil program;
Teaching methods include	demonstration, brainstorming, and practical work.
Teaching aids:	Blackboard, electronic whiteboard, assignments, software.
Teaching methods:	Individual and group
Teaching conditions:	Electronic whiteboard and projector in the auditorium.
Monitoring and evaluation	Observation, oral assessment, question and answer, and computer-based practical work.

15-practical lesson: Dynamic econometric models

Reja:

1. **Funksional, statistik va korrelyatsion bog'lanishlar. Shartli o'rtacha.**
2. **Regrissiya tenglamasi. Chiziqli regrissiyaning tanlanma tenglamasi.**
3. **Korrelyatsiya koeffitsiyenti. Korrelyatsiya koeffitsoyentini hisoblash formulalari.**
4. **Korrelyatsiya koeffitsiyentining xossalari. Chiziqsiz regrissiya. Korrelyatsion nisbat.**

Practical Lesson: Dynamic Econometric Models

Objectives

•Understand dynamic econometric models.
• Learn lag variables and autoregressive models.
• Solve practical econometric problems using formulas.

1. Theoretical Background

Dynamic econometric models explain relationships where current values depend on past values. They are widely used in economics for forecasting and policy analysis.

Main formulas

Autoregressive model AR(1): $Yt = \alpha + \beta Y(t-1) + \varepsilon t$
Distributed lag model: $Yt = \alpha + \beta 0Xt + \beta 1X(t-1) + \varepsilon t$
Adaptive expectations model: $Yt = \alpha + \beta Xt + \lambda Y(t-1) + \varepsilon t$
Growth rate: $g = [(Yt - Y(t-1))/Y(t-1)] \times 100\%$

2. Solved Problem 1

Suppose a dynamic model is:
$Yt=10+0.8Y(t-1)$
If $Y(t-1)=50$, find Yt.
Solution:
$Yt=10+0.8(50)$
$Yt=10+40$
$Yt = 50$

3. Solved Problem 2

Suppose a distributed lag model:
$Yt=5+2Xt+0.5X(t-1)$
If $Xt=20$ and $X(t-1)=10$, find Yt.
Solution:
$Yt=5+2(20)+0.5(10)$
$Yt=5+40+5$
$Yt = 50$

4. Practice Tasks

1. $Yt = 8 + 0.7Y(t-1)$, $Y(t-1)=30$
2. $Yt = 3 + 1.5Xt + 0.2X(t-1)$, $Xt=15$, $X(t-1)=5$
3. Calculate growth rate if $Y(t-1)=100$ and $Yt=120$

Conclusion

Dynamic econometric models are useful for understanding time-dependent economic relationships and forecasting future values.

$$\frac{\partial S}{\partial a} = -2\sum_{i=1}^{n} y_i + 2na + 2b\sum_{i=1}^{n} x_i\,,$$

$$\frac{\partial S}{\partial b} = -2\sum_{i=1}^{n} y_i x_i + 2a\sum_{i=1}^{n} x_i + 2b\sum_{i=1}^{n} x_i^2$$

$$\begin{cases} \dfrac{\partial S}{\partial a} = 0, \\ \dfrac{\partial S}{\partial b} = 0. \end{cases} \Rightarrow \begin{cases} -2\sum_{i=1}^{n} y_i + 2na + 2b\sum_{i=1}^{n} x_i = 0, \\ -2\sum_{i=1}^{n} y_i x_i + 2a\sum_{i=1}^{n} x_i + 2b\sum_{i=1}^{n} x_i^2 = 0. \end{cases}$$

We obtain the following system of normal equations by applying elementary transformations to the final system of equations

$$\begin{cases} na + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i\,, \\ a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} y_i x_i\,. \end{cases}$$

From this system of equations, the parameters a and b can be determined

$$a = \frac{(\sum x_i^2)\cdot(\sum y_i\,) - (\sum x_i\,)\cdot(\sum x_i y_i\,)}{n\cdot(\sum x_i^2) - (\sum x_i\,)^2},$$

$$b = \frac{n\cdot(\sum x_i y_i\,) - (\sum x_i\,)\cdot(\sum y_i\,)}{n\cdot(\sum x_i^2) - (\sum x_i\,)^2}.$$

We denote the obtained parameter values as a0a_0a0 and b0b_0b0, respectively. For these values of a0a_0a0 and b0b_0b0, the condition

∑i=1nεi2→min⁡\sum_{i=1}^{n}\varepsilon_i^2 \rightarrow \mini=1∑nεi2→min

is satisfied.

In the linear regression equation, the parameter bbb is called the regression coefficient. Its value shows how much the dependent variable changes on average when the explanatory variable changes by one unit.

In the regression equation, the parameter aaa represents the value of yyy when x=0x = 0x=0. If a<0a < 0a<0, then at x=0x = 0x=0, the parameter aaa has no economic meaning.

The regression equation is always supplemented by an indicator of the strength of the relationship between variables. For this purpose, the correlation coefficient is

$$r_{xy} = \frac{\sum(x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \cdot \sum(y_i - \bar{y})^2}}$$

or

$$r_{xy} = \frac{n\sum x_i y_i - \sum x_i \cdot \sum y_i}{\sqrt{[n\sum x_i^2 - (\sum x_i)^2] \cdot [n\sum y_i^2 - (\sum y_i)^2]}}.$$

r_{xy}	*0,1-0,3*	*0,3-0,5*	*0,5-0,7*	*0,7-0,9*	*0,9* va undan yuqori
Bog’lanish zichligi darajasi	bo’sh	o’rta miyona	sezilarli	yuqori	juda ham yuqori

1. $y = a_0 + a_1 x$

$$\begin{cases} na_0 + a_1\sum x = \sum y, \\ a_0\sum x + a_1\sum x^2 = \sum y \cdot x. \end{cases} \quad (1.1)$$

2. $y = a_0 + a_1 x + a_2 x^2$

$$\begin{cases} na_0 + a_1\sum x + a_2\sum x^2 = \sum y, \\ a_0\sum x + a_1\sum x^2 + a_2\sum x^3 = \sum y \cdot x, \\ a_0\sum x^2 + a_1\sum x^3 + a_2\sum x^4 = \sum y \cdot x^2. \end{cases} \quad (1.2)$$

3. $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$$\begin{cases} na_0 + a_1\sum x + a_2\sum x^2 + a_3\sum x^3 = \sum y, \\ a_0\sum x + a_1\sum x^2 + a_2\sum x^3 + a_3\sum x^4 = \sum y \cdot x, \\ a_0\sum x^2 + a_1\sum x^3 + a_2\sum x^4 + a_3\sum x^5 = \sum y \cdot x^2, \\ a_0\sum x^3 + a_1\sum x^4 + a_2\sum x^5 + a_3\sum x^6 = \sum y \cdot x^3. \end{cases} \quad (1.3)$$

4. $k - y = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^k$

$$\begin{cases} na_0 + a_1\sum x + a_2\sum x^2 + \ldots + a_n\sum x^k = \sum y, \\ a_0\sum x + a_1\sum x^2 + a_2\sum x^3 + \ldots + a_n\sum x^{k+1} = \sum y \cdot x, \\ \Lambda \\ a_0\sum x^k + a_1\sum x^{k+1} + a_2\sum x^{k+2} + \ldots + a_n\sum x^{2k} = \sum y \cdot x^k. \end{cases} \quad (1.4)$$

5. $y = a_0 + \frac{a_1}{x}$

$$\begin{cases} na_0 + a_1\sum \frac{1}{x} = \sum y, \\ a_0\sum \frac{1}{x} + a_1\sum \frac{1}{x^2} = \sum \frac{y}{x}. \end{cases} \quad (1.5)$$

6. $k - y = a_0 + \frac{a_1}{x^k}$

$$\begin{cases} na_0 + a_1 \sum \frac{1}{x^k} = \sum y, \\ a_0 \sum \frac{1}{x^k} + a_1 \sum \frac{1}{x^{2k}} = \sum \frac{y}{x^k}. \end{cases} \quad (1.6)$$

7. $y = a_0 \cdot a_1^x$

$$\begin{cases} n \ln a_0 + \ln a_1 \sum x = \sum \ln y, \\ \ln a_0 \sum x + \ln a_1 \sum x^2 = \sum x \cdot \ln y. \end{cases} \quad (1.7)$$

8. $y = a_0 x^{a_1}$

$$\begin{cases} n \ln a_0 + a_1 \sum \ln x = \sum \ln y, \\ \ln a_0 \sum \ln x + a_1 \sum \ln^2 x = \sum \ln y \cdot \ln x. \end{cases} \quad (1.8)$$

9. $\ln y = a_0 + a_1 x$

$$\begin{cases} na_0 + a_1 \sum x = \sum \ln y, \\ a_0 \sum x + a_1 \sum x^2 = \sum x \cdot \ln y. \end{cases} \quad (1.9)$$

10. $y = a_0 + a_1 \ln x$

$$\begin{cases} na_0 + a_1 \sum \ln x = \sum y, \\ a_0 \sum \ln x + a_1 \sum \ln^2 x = \sum y \cdot \ln x. \end{cases} \quad (1.10)$$

11. $y = \frac{a_0}{1 + a_1 \cdot e^{-bx}}$

$$\begin{cases} a_0 \sum \frac{1}{y^2} + a_1 \cdot \left(-\sum \frac{e^{-bx}}{y} \right) = \sum \frac{1}{y}, \\ a_0 \cdot \left(-\sum \frac{e^{-bx}}{y} \right) + a_1 \cdot \sum e^{-2bx} = \sum e^{-bx}. \end{cases} \quad (1.11)$$

12.

$$y = a_0 \cdot x_1^{a_1} \cdot x_2^{a_2} \ (a_1 + a_2 \le 1).$$

$$\ln y = \ln a_0 + a_1 \ln x_1 + a_2 \ln a_2.$$

$$\begin{cases} n \ln a_0 + a_1 \sum \ln x_1 + a_2 \sum \ln x_2 = \sum \ln y, \\ \ln a_0 \sum \ln x_1 + a_1 \sum \ln^2 x_1 + a_2 \sum \ln x_1 \cdot \ln x_2 = \sum \ln x_1 \cdot \ln y, \\ \ln a_0 \sum \ln x_2 + a_1 \sum \ln x_1 \cdot \ln x_2 + a_2 \sum \ln^2 x_2 = \sum \ln x_2 \cdot \ln y. \end{cases} \quad (1.12)$$

$$y = a_0 + a_1 x_1 + a_2 x_2 \quad (2.16)$$

a_0, a_1, a_2:

$$\begin{cases} na_0 + a_1\sum x_1 + a_2\sum x_2 = \sum y, \\ a_0\sum x_1 + a_1\sum x_1^2 + a_2\sum x_1x_2 = \sum yx_1, \\ a_0\sum x_2 + a_1\sum x_1x_2 + a_2\sum x_2^2 = \sum yx_2. \end{cases} \quad (2.17)$$

$$R_{yx_1x_2} = \sqrt{\frac{r_{yx_1}^2 + r_{yx_2}^2 - 2r_{yx_1}r_{yx_2}r_{x_1x_2}}{1 - r_{x_1x_2}^2}} \quad (2.18)$$

r_{yx_1}, r_{yx_2}, $r_{x_1x_2}$ -

$$\begin{cases} r_{yx_k} = \dfrac{\overline{x_k \cdot y} - \bar{x}_k \cdot \bar{y}}{\sigma_{x_k} \cdot \sigma_y}, \\ r_{x_1x_2} = \dfrac{\overline{x_1 \cdot x_2} - \bar{x}_1 \cdot \bar{x}_2}{\sigma_{x_1} \cdot \sigma_{x_2}}, \end{cases} \quad (k = 1; 2) \quad (2.19)$$

$$\begin{cases} \sigma_{x_k} = \sqrt{\dfrac{1}{n}\sum_{i=1}^{n}\left(x_k^{(i)} - \bar{x}_k\right)^2} = \sqrt{\bar{x}_k^2 - (\bar{x}_k)^2}, \\ \sigma_y = \sqrt{\dfrac{1}{n}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2} = \sqrt{\bar{y}^2 - (\bar{y})^2}. \end{cases} \quad (k = 1; 2) \quad (2.20)$$

N	X	Y	XY	X2	Y2	Yx	Y-Yx	Ai
1	10,5	8,12	85,26	110,25	65,9344	8,2515	-0,1315	1,619458
2	11,6	10	116	134,56	100	9,1348	0,8652	8,652
3	12,3	8,41	103,443	151,29	70,7281	9,6969	-1,2869	15,30202
4	13,7	12,1	165,77	187,69	146,41	10,8211	1,2789	10,56942
5	14,5	12,4	179,8	210,25	153,76	11,4635	0,9365	7,552419
6	16,1	11,4	183,54	259,21	129,96	12,7483	-1,3483	11,82719
7	17,3	12,8	221,44	299,29	163,84	13,7119	-0,9119	7,124219
8	18,7	13,9	259,93	349,69	193,21	14,8361	-0,9361	6,734532
9	20,1	17,3	347,73	404,01	299,29	15,9603	1,3397	7,743931
10	21,8	17,5	381,5	475,24	306,25	17,3254	0,1746	0,997714
Jami	156,6	123,93	2044,413	2581,48	1629,383	123,9498	0	78,12291
O'rtacha	15,66	12,393	204,4413	258,148	162,9383			7,812291
σ	3,593383	3,058071						
σ^2	12,9124	9,351801						

$$\sigma_x^2 = \overline{x^2} - \bar{x}^2$$
$$\sigma_y^2 = \overline{y^2} - \bar{y}^2$$

N	X	Y	XY	X2	Y2	Yx	Y-Yx	(Y-Yx)^2	Ai
1	1,021189	0,909556	0,928829	1,042828	0,827292	8,264448	-0,14445	0,020865	1,778915
2	1,064458	1	1,064458	1,133071	1	9,133886	0,866114	0,750153	8,661135
3	1,089905	0,924796	1,00794	1,187893	0,855248	9,687339	-1,27734	1,631596	15,18834
4	1,136721	1,082785	1,230824	1,292134	1,172424	10,79462	1,305383	1,704025	10,78829
5	1,161368	1,093422	1,269865	1,348776	1,195571	11,42755	0,972447	0,945652	7,842311
6	1,206826	1,056905	1,2755	1,456429	1,117048	12,69384	-1,29384	1,674018	11,34946
7	1,238046	1,10721	1,370777	1,532758	1,225914	13,64389	-0,84389	0,712143	6,592858
8	1,271842	1,143015	1,453734	1,617581	1,306483	14,75261	-0,85261	0,726938	6,13386
9	1,303196	1,238046	1,613417	1,69832	1,532758	15,86166	1,43834	2,068823	8,314105
10	1,338456	1,243038	1,663752	1,791466	1,545144	17,20878	0,291219	0,084809	1,664109
Jami	11,83201	10,79877	12,8791	14,10125	11,77788	123,4686	0	10,31902	78,31338
O'rtacha	1,183201	1,079877	1,28791	1,410125	1,177788			1,031902	7,831338
σ	0,100804	0,10795							
σ^2	0,010162	0,011653							

Foydalanilgan adabiyotlar

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