**O’ZBEKISTON RESPUBLIKASI OLIY TA’LIM, FAN VA INNOVATSIYALAR VAZIRLIGI**

**Samarqand Davlat Veterinariya Meditsinasi, Chorvachilik va Biotexnologiyalar Universiteti**

**Axborot texnologiyalar va tabiiy fanlar kafedrasi**

o’qituvchisi Xamitov Shoxzod Normurodovichning Agrotexnologiya fakulteti O‘rmonchilik va aholi yashash joylarini ko‘kalamzorlashtirish ta’lim yo‘nalishi 1-bosqich 107-guruh talabalari uchun “Oliy matematika” fanidan “**Aniqmas integrallar va ularning hisoblash usullari**” mavzusidagi

AMALIY MASHG’ULOTI ISHLANMASI

# Mavzu: “Aniqmas integrallar va ularning hisoblash usullari”

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## 6-mavzu. Aniqmas integrallar va ularning hisoblash usullari

**Reja:**

1. Aniqmas integral.

2. Integrallash usullari: bevosita, o‘zgaruvchilarni almashtirish va bo‘laklab integrallash.

Tayanch iboralar:boshlang‘ich funksiya,aniqmas integral,aniq integral, bevosita, o‘zgaruvchilarni almashtirish va bo‘laklab integrallash.

## Mavzu: Aniqmas integrallar va ularning hisoblash usullari

### Amaliy mashg‘ulot texnologiyasi

| | |
|---|---|
| **Vaqti – 80 minut** | O‘quvchilar soni: 23 nafar |
| **O‘quv mashg‘ulotining shakli** | Amaliy (Misol, masalalar yechishga o‘rgatish) |
| **Mashg‘ulotning rejasi** | 1. O‘tgan mavzuga doir misollar yechish<br>2.Integrallash usullari: bevosita, o‘zgaruvchilarni almashtirish va bo‘laklab integrallash.<br>3. Irratsional ifodalar va trigonometrik funksiyalarni integrallash.<br>4. Nyuton- Leybnis formulasi. Aniq integrallarni hisoblash usullari.<br>5. Misol va masalalar yechish |
| **O‘quv mashg‘ulotining maqsadi: a) <u>ta’limiy</u>**- Funksiyadan integral olishni bilish. Integrallar jadvalidan foydalanib misollarga qo‘llay olish.<br>**b) <u>tarbiyaviy</u>** - o‘quvchilarning kasbiy bilimlari – *ta’lim* sohasiga tegishli bilimlardan foydalanib, o‘rganilayotgan mavzuga qiziqish o‘yg‘otish, matematikaning hayotdagi, kasblardagi o‘rnini ko‘rsatish orqali ularni mehnatsevarlikka, diqqatni jamlashga, fikrlashga o‘rgatish, matematik tafakkurni shaklantirish.<br>**v) <u>rivojlantiruvchi</u>** - taqqoslash, umumlashtirish, xulosa chiqarish usullarini qo‘llash ko‘nikmasini shakllantirish; | |
| **Tayanch so‘z va iboralar** | boshlang‘ich funksiya,aniqmas integral,aniq integral, bevosita, o‘zgaruvchilarni almashtirish va bo‘laklab integrallash. |
| **Pedagogik vazifalar:**<br>- Boshlang‘ich funksiya, aniqmas integral va aniq integral. haqida ma’lumot berish;<br>- Aniqmas integral jadvalidan foydalanishni bilish;<br>- Nyuton-Leybnis formulasidan foydalana olish;<br>- Formulalarni misollar ishlashda qo‘llashni bilish; | **O‘quv faoliyatining natijalari:**<br>**O‘quvchi**:<br>- Boshlang‘ich funksiya, aniqmas integral va aniq integrallar haqida gapirib beradi;<br>- Aniqmas integral jadvalidagi formulalarini qo‘llay oladi.<br>-Formulalardan misollar ishlashda foydalana oladi; |
| O‘qitish uslubi va texnikasi | Kichik guruhlarda ishlash, modelli o‘qitish usuli, klaster, test. |
| O‘qitish vositalari | Ma’ruzalar matni, rangli plakatlar, mavzu bo‘yicha tarqatma materiallar. |

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| O‘qitish shakllari | *Mustaqil ish, guruhlarda ishlash, jamoada ishlash. |
| O‘qitish sharoitlari | Guruhdagi ishlarni tashkillashtirish uchun muvofiqlashgan texnik uskunalar bilan jihozlangan auditoriya. |
| O‘quvchilarning berilgan o‘quv mashg‘ulotlari uchun kerakli bilim va ta’lim mahoratlari ro‘yxati. | 1.Integrallash usullari: bevosita, o‘zgaruvchilarni almashtirish va bo‘laklab integrallash.<br>2. Irratsional ifodalar va trigonometrik funksiyalarni integrallash.<br>3. Nyuton- Leybnis formulasi. Aniq integrallarni hisoblash usullari. |

## Amaliy mashg‘ulotning texnologik xaritasi

| **Bosqichlar vaqti** | **Faoliyat mazmuni** | |
|---|---|---|
| | **O‘qituvchi** | **O‘quvchi** |
| **1-bosqich**<br>**Kirish**<br>**(10 min)** | 1.1.O‘tilgan mavzular bo‘yicha tarqatma materiallar tarqatiladi.<br>1.2. O‘quv mashg‘ulotining mavzu va rejasini ma’lum qiladi. Erishadigan natijalar bilan tanishtiradi. Mazkur mashg‘ulot muammoli tarzda o‘tishini e’lon qiladi. | 1.1. Eshitadilar va yozib oladilar. |
| **2-bosqich**<br>**Asosiy**<br>**(55 min)** | 2.1. O‘quvchilar e’tiborini rejadagi savollar va ulardagi tushunchalarga qaratadilar. Kichik guruhlarga bo‘linadi.<br>2.2. Muammoli savollarni o‘rtaga tashlaydi va ularni birgalikda o‘qishga chorlaydi<br>2.3. Mavzuni mustahkamlash uchun klaster tuzushadi.<br>2.4. Mavzuga doir misollar ishlashadi.<br>1.Integrallash usullari: bevosita, o‘zgaruvchilarni almashtirish va bo‘laklab integrallash.<br>2. Irratsional ifodalar va trigonometrik funksiyalarni integrallash.<br>3. Nyuton- Leybnis formulasi. Aniq integrallarni hisoblash usullari.<br>2.5.Mustaqil ishlash uchun misollar beriladi. | 2.1. O‘quvchilar javob beradilar, daftarlariga chizadilar, jadvalnning 1 va 2 ustunlarini to‘ldiradilar.<br>2.2. Muammoga e’tiborni qaratadilar va yozib oladilar.<br>2.3. Yozib oladilar va o‘z bilimlari bilan solishtiradilar.<br>2.4. Muammo yuzasidan o‘z yechimlarini taklif qiladilar. Munozara qiladilar. Javob beradilar.<br>2.5. Optimal yechimlar yuzasidan takliflar beradilar. |

| | | |
|---|---|---|
| **3-bosqich**<br>**Yakuniy**<br>**(15 min)** | 3.1. Mavzuga xulosa qiladi.<br>3.2. Rejadagi natijaga erishishda faol ishtirokchilarni rag‘batlantiradi.<br>3.3. Mustaqil ish uchun vazifa beradi: «Integral hisob asoslari» mavzusini o‘rganish.<br>3.4. Uyda vazifa uchun misollar beriladi. | 3.1. Eshitadilar. Yozib oladilar.<br>3.2.Yozib oladilar va uyda ishlaydilar. |

## ANIQMAS INTEGRAL

**Ta’rif.** Agar ***F(x)*** funksiyaning hosilasi ***f(x)*** funksiyaga teng, ya’ni ***F′(x)=f(x)*** bo’lsa, u holda ***F(x)*** funksiyaga ***f(x)*** funksiyaning boshlang‘ich funksiyasi deyiladi.

Ta’rif. *f(x)* funksiyaning barcha boshlang‘ich funksiyalaridan iborat {*F(x)*+*c*} to‘plamni *f(x)* funksiyaning ANIQMAS INTEGRALI deyiladi va

$$\int f(x)dx = F(x) + c$$

ko‘rinishda yoziladi. *c* – o‘zgarmas son

## INTEGRALLASH QOIDALARI

$$I. \quad \int f(x)dx = F(x) + c$$

$$II. \int kf(x)dx = k\int f(x)dx + c. \quad k - const., \quad k \neq 0$$

$$III. \quad \int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx + c$$

$$IV. \quad \int dF(x) = F(x) + c., \quad d[\int f(x)dx] = f(x)dx.$$

## ASOSIY INTEGRALLAR JADVALI

1. $\int dx = x + c;$
2. $\int x^m dx = \frac{x^{m+1}}{m+1} + c(m \neq -1);$
3. $\int a^x dx = \frac{a^x}{\ln a} + c \quad (a > 0; a \neq 1);$
4. $\int e^x dx = e^x + c;$
5. $\int \frac{dx}{x} = \ln|x| + c;$
6. $\int \sin x dx = -\cos x + c;$
7. $\int \cos dx = \sin x + c;$
8. $\int \frac{dx}{\sin^2 x} = -ctgx + c;$
9. $\int \frac{dx}{\cos^2 x} = tgx + c;$
10. $\int tgxdx = -\ln|\cos x| + c.$
11. $\int ctgxdx = \ln|\sin x| + c.$
12. $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + c.$
13. $\int \frac{dx}{1+x^2} = arctgx + c;$
14. $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + c.$

Misol 1. $\int (x+1)^2 dx$ ni integrallang.

Yechilishi.

$$\int (x+1)^2 dx = \int (x^2+2x+1)dx = \int x^2 dx + 2\int x dx + \int dx = \frac{x^{2+1}}{2+1} + 2\cdot\frac{x^{1+1}}{1+1} + x + c =$$

$$= \frac{x^3}{3} + x^2 + x + c$$

Misol 2. $\int (\cos 3x + 2e^x + \frac{3}{\sin^2 x})dx$ ni integrallang.

Yechilishi.

$$\int (\cos 3x + 2e^x + \frac{3}{\sin^2 x})dx = \int \cos 3x \cdot dx + 2\int e^x dx + 3\int \frac{dx}{\sin^2 x} = \frac{1}{3}\sin 3x + 2e^x - 3ctgx + c.$$

Misol 3. $\int (x^2+5)^3 dx$ ni integrallang.

Yechilishi.

$$\int (x^2+5)^3 dx = \int [(x^2)^3 + 3(x^2)^2 * 5 + 3x^2 * 5^2 + 5^3]dx = \int (x^6 + 15x^4 + 75x^2 + 125)dx =$$

$$= \int x^6 dx + \int 15x^4 dx + \int 75x^2 dx + \int 125 dx = \int x^6 dx + 15\int x^4 dx + 75\int x^2 dx + 125\int dx =$$

$$= \frac{x^{6+1}}{6+1} + 15\frac{x^{4+1}}{4+1} + 75\frac{x^{2+1}}{2+1} + 125x + c = \frac{x^7}{7} + 3x^5 + 25x^3 + 125x + c.$$

Misol 4. $\int \cos 4x dx$ ni integrallang.

Yechilishi. $\int \cos 4x dx = \frac{1}{4}\sin 4x + c.$

## ANIQMAS INTEGRALDA O‘ZGARUVCHINI ALMASHTIRISH

$$\int f(x)dx = \int f(\varphi(t))\,\varphi'(t)\cdot dt = F(\varphi(t)) + c$$

Misol 5. $\int (2+3x)^5 dx$ ni integrallang.

Yechilishi.

$$\int (2+3x)^5 dx = \left| \begin{array}{l} 2+3x = t \Rightarrow 3x = t-2 \Rightarrow \\ \Rightarrow x = \frac{t-2}{3} \Rightarrow x = \frac{1}{3}\cdot t - \frac{2}{3} \Rightarrow \\ \Rightarrow dx = \frac{1}{3}dt \end{array} \right| = \int t^5 \cdot \frac{1}{3}\cdot dt = \frac{1}{3}\int t^5 \quad dt = \frac{1}{3}\cdot\frac{t^{5+1}}{5+1} + c = \frac{t^6}{18} + c = \frac{(2+3x)^6}{18} + c.$$

Misol 6. $\int (x-3)^5 dx$ **ni integrallang.**

Yechilishi.

$$\int (x-3)^5 dx = \left| \begin{array}{l} t = x-3; \\ x = t+3; \\ dx = dt \end{array} \right| = \int t^5 dt = \frac{t^{5+1}}{5+1} + c = \frac{(x-3)^6}{6} + c.$$

Misol 7. $\int \cos(5x-3)dx$ **ni integrallang.**

Yechilishi.

$$\int \cos(5x-3)dx = \left| \begin{array}{l} t = 5x-3 \\ 5x = t+3; \\ x = \frac{1}{5}t + \frac{3}{5}; \\ dx = \frac{1}{5}dt. \end{array} \right| = \int \cos t * \frac{1}{5}dt = \frac{1}{5}\int \cos tdt = \frac{1}{5}\sin t + c = \frac{1}{5}\sin(x-3) + c.$$

## ANIQMAS INTEGRALLARNI BO‘LAKLAB INTEGRALLASH

*u=u(x)* va *v=v(x)* funksiyalarning uzluksiz *u/(x)* va *v/(x)* hosilalari mavjud bo’lsin.
U holda ko’paytmadan hosila olish qoidasiga ko’ra,

$$d(u\cdot v) = udv + vdu;$$

$$\int u\cdot dv = \int d(u\cdot v) - \int v\cdot du;$$

$$\int u\cdot dv = u\cdot v - \int vdu;$$

Misol 8. $\int xe^x dx$ **ni integrallang.**

Yechilishi.

$$\int x\ e^x dx = \left| \begin{array}{l} u = x \Rightarrow dx = du; \\ dv = e^x dx; \int dv = \int e^x dx; \\ v = e^x. \end{array} \right| = x\cdot e^x - \int e^x dx = x\cdot e^x - e^x + c.$$

Misol 9. $\int \ln xdx$ **ni integrallang.**

Yechilishi.

$$\int ln\,xdx = \left| \begin{array}{l} u = ln\,x;\ du = \frac{1}{x}dx; \\ dv = dx; \quad \int dv = \int dx; \\ v = x. \end{array} \right| = x\cdot ln\,x - \int x\cdot\frac{1}{x}\cdot dx = x\,ln\,x - x + c.$$

Misol 10. $\int x\cos xdx$ **ni integrallang.**

Yechilishi.

$$\int x\cos xdx = \left| \begin{array}{l} u = x \Leftrightarrow du = dx; \\ dv = \cos xdx; \\ \int dv = \int \cos xdx; \\ v = \sin x \end{array} \right| = x * \sin x - \int \sin xdx = x\sin x - (-\cos x) + c =$$

$$= x\sin x + \cos x + c.$$

## MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi integrallarni toping:

**1.** $\int\left(x^3+2x+\frac{4}{x}\right)dx$; Javobi: $\frac{x^4}{4}+x^2+4\ln x+c$.

**2.** $\int(1-\frac{1}{\cos^2 3x})dx$; Javobi: $x-\frac{1}{3}tg3x+c$.

**3.** $\int\frac{x^2+2x+2}{\sqrt{x^3}}dx$; Javobi: $\frac{2}{3}\sqrt{x^3}+4\sqrt{x}-\frac{4}{\sqrt{x}}+c$.

**4.** $\int 2\sin 3x dx$; Javobi: $-\frac{2}{3}\cos 3x+c$.

**5.** $\int(\frac{1}{3}x^2-6x+\frac{3}{4})dx$; Javobi: $\frac{x^3}{9}-3x^2+\frac{3}{4}x+c$

**6.** $\int\sqrt[3]{x^2}dx$; Javobi: $\frac{3}{5}\sqrt[3]{x^5}+c.$

**7.** $\int 3\cos 3x dx$. Javobi: $\sin 3x+c.$

Aniqmas integralda o‘zgaruvchini almashtirishga doir misollar:

**8.** $\int\sqrt{2x}dx;$ Javobi: $\frac{1}{3}\sqrt{(2x)^3}+c.$

**9.** $\int\sqrt{3x+5}dx;$ Javobi: $\frac{2}{9}\sqrt{(3x+5)^3}+c.$

**10.** $\int\sin(5x-3)dx;$ Javobi: $-\frac{1}{5}\cos(5x-3)+c.$

**11.** $\int\cos(16x+5)dx;$ Javobi: $\frac{1}{16}\sin(16x+5)+c.$

**12.** $\int\frac{xdx}{x^2+1};$ Javobi: $\frac{1}{2}\ln(x^2+1)+c.$

**13.** $\int\ln(4x-6)dx;$ Javobi: $\frac{4}{4x-6}+c.$

Aniqmas integralni bo‘laklab integrallashga doir misollar:

**14.** $\int x\sin x dx$; Javobi: $-x\cos x-\sin x+c.$

**15.** $\int x\cos 2x dx;$ Javobi: $\frac{1}{2}x\sin 2x+\cos 2x+c.$

**16.** $\int(2x+1)\sin 3x dx;$ Javobi: $(2x+1)(-\frac{1}{3}\cos 3x)+\frac{2}{9}\sin 3x+c.$

**17**. $\int xarctgxdx;$ Javobi: $\frac{x^2}{2}arc\,tgx - \frac{x}{2} + \frac{1}{2}arctgx + c.$

## IRRATSIONAL IFODALAR VA TRIGONOMETRIK FUNKSIYALARNI INTEGRALLASH.

### Ba'zi irratsional funktsiyalarni integrallash

1. $R(x,\sqrt[m]{\frac{ax+b}{cx+d}}$ *ko'rinishdagi ifodalarni integrallash.*

2. $x^m\left(a+bx^n\right)^P$ *ko'rinishdagi ifodalarni integrallash.*

*3.* $R(x,\sqrt{ax^2+bx+c}$ *ko'rinishdagi ifodalarni integrallash.*

$R\left(x,\sqrt[m]{\frac{ax+b}{cx+d}}\right)dx$ *ko'rinishdagi ifodalarni integrallash.*

Bunday ifodalarni integrallashda integral ostidagi ifodani ratsinonal ifoda ko'rinishiga keltirish lozim. Ratsional ifoda shakliga keltiradigan almashtirishni aniqlash kerak.

Bunda $t=\varphi(x)=\sqrt[m]{\frac{ax+b}{cx+d}}$ (1)

ko'rinishdagi almashtirish olinadi.

Misol sifatida quyidagi:

$$\int R\left(x,\sqrt[m]{\frac{ax+b}{cx+d}}\right)dx \qquad (2)$$

ifodani qaraylik. Bunda *R* - ikkita argumentning ratsional funksiyasi, *m*-natural son, *a*, *b*, *c* va *d* lar o'zgarmas sonlardan iborat. $t=f\left(x\right)=\sqrt[m]{\frac{ax+b}{cx+d}}$ almashtirish olamiz, bunda

$$t^m=\frac{ax+b}{cx+d}, \quad x=g\left(t\right)=\frac{dt^m-b}{a-ct^m}$$

bo'lsin. U holda, (2) integral

$$\int R\left(\varphi(t),\varphi'(t)\right)dt \qquad (3)$$

ko‘rinishga keladi. $R, \varphi, \varphi'$ lar ratsional funksiyalardir. (3) integralni oldingi mavzudagi qoida asosida hisoblab, $t = f(x)$ ni qo‘yib, so‘ngra eski o‘zgaruvchiga qaytiladi.

***Misol.*** $\int \frac{dx}{\sqrt[3]{(x-1)(x+1)^2}} = \int \sqrt[3]{\frac{x+1}{x-1}} \cdot \frac{1}{x+1} dx$

(1) almashtirishni amalga oshiramiz:

$$t = \sqrt[3]{\frac{x+1}{x-1}}, \quad x = \frac{t^3+1}{t^3-1}, \; dx = -\frac{6t^2}{\left(t^3-1\right)^2}dt.$$

U holda,

$$\int \sqrt[3]{\frac{x+1}{x-1}} \cdot \frac{1}{x+1}dx = \int \frac{-3}{t^3-1}dt = \int\left(-\frac{1}{t-1} + \frac{t+2}{t_2+t+1}\right)dt = \int -\frac{1}{t-1}dt + \int \frac{t+2}{t^2+t+1}dt =$$

$$= \frac{1}{2}\ln\frac{t^2+t+1}{(t-1)^2} + \sqrt{3}arctg\frac{2t+1}{\sqrt{3}} + c.$$

Bundan, eski o‘zgaruvchiga qaytiladi.

## $x^m\left(a+bx^n\right)^p$ ko‘rinishdagi ifodani integrallash

Agar $p$ butun son bo‘lsa, ifoda oldingi o‘rganilgan guruhga kiradi. Qaralayotgan ifodani integrallash uchun

$$z = x^n \qquad (4)$$

almashtirish olamiz va ifoda shaklini o‘zgartiramiz:

$$x^m\left(a+bx^n\right)^p dx = \frac{1}{n}(a+bz)^p \cdot z^{\frac{m+1}{n}-1}dz.$$

Bundan $\frac{m+1}{n} - 1 = q$ desak,

$$\int x^m\left(a+bx^n\right)^p dx = \frac{1}{n}\int (a+bz)^p z^q dz. \qquad (5)$$

Agar *q* butun son bo‘lsa, oldin o‘rganilgan misol turiga kelinadi.

(2) ning o‘ng tomonidagi integralni quyidagi ko‘rinishda yozamiz:

$$\int\left(\frac{a+bz}{z}\right)z^{p+q}dz.$$

Bundagi $p+q$ butun son bo‘lganda yana o‘rganilgan misolga kelamiz.

***1-misol.*** Integralni hisoblang:

$$\int\frac{x^3}{\sqrt{x^3+2}}dx.$$

***Yechilishi*:** $x=t^6$ almashtirish olamiz. U holda, $dx=6tdt$ bo‘ladi. Integrallashni amalga oshiramiz:

$$\int\frac{x^3}{\sqrt{x^3+2}}dx=\int\frac{dx}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}=\int\frac{6t^5dt}{\left(1+t^2\right)t^3}=6\int\frac{t^2}{1+t^2}dt=6\int\frac{t^2+1-1}{t^2+1}dt=$$

$$=6\int\left(1-\frac{1}{1+t^2}\right)dt=6\int dt-6\int\frac{dt}{1+t^2}=6\left(t-arctgt\right)+c=6\left(\sqrt[6]{x}-arctg\sqrt[6]{x}\right)+c.$$

***2-misol.*** Quyidagi integralni hisoblang:

$$\int x\sqrt{a-x}dx.$$

***Yechilishi*:** $a-x=t^2$ belgilash kiritamiz. Bundan, $x=a-t^2$, $dx=-2tdt,$ $\sqrt{a-x}=t.$ U holda,

$$\int x\sqrt{a-x}dx=-2\int\left(a-t^2\right)t^2dt=-2a\int t^2dt+2\int t^4dt=-2a\cdot\frac{t^3}{3}+2\cdot\frac{t^5}{5}+c=$$

$$=-\frac{2a}{3}\left(\sqrt{a-x}\right)^3+\frac{2}{5}\left(\sqrt{a-x}\right)^5+c.$$

## $R\left(x,\sqrt{ax^2+bx+c}\right)$ ko‘rinishidagi ifodalarni integrallash. Eyler almashtirishlari

Har doim ham kvadrat uchhad teng ildizlarga ega bo‘lmaydi. Shuning uchun ulardan olingan ildizning ratsional ifoda bilan almashtirishi mumkin bo‘lmaydi. Bunday hollarda Eylerning uchta almashtirishi yordamida integral ostidagi ifoda ratsional ifodaga keladi.

$a>0$ bo’lsa, *birinchi almashtirish*

$$\sqrt{ax^2+bx+c}=t-\sqrt{a}x. \qquad (6)$$

qo‘llaniladi. Uning ikkala tomonini kvadratga ko‘taramiz:

$$bx+c=t^2-2\sqrt{a}tx.$$

U holda,

$$x=\frac{t^2-c}{2\sqrt{a}t+b},\ \sqrt{ax^2+bx+c}=\frac{\sqrt{a}t^2+bx+c\sqrt{a}}{2\sqrt{a}t+b},\ dx=2\cdot\frac{\sqrt{a}t^2+bt+c\sqrt{a}}{\left(2\sqrt{a}t+b\right)^2}dt.$$

Topilgan ifodalarni

$$\int R\left(x,\sqrt{ax^2+bx+c}\right) \qquad (7)$$

integralga qo‘yilsa, u holda, qaralayotgan misol $t$ ning ratsional funksiyasini integrallashga keltiriladi. Keyin eski $x$ o‘zgaruvchiga qaytiladi,

$$t=\sqrt{ax^2+bx+c}+\sqrt{a}x.$$

$c>0$ bo‘lsa, *ikkinchi almashtirish*

$$\sqrt{ax^2+bx+c}=xt+\sqrt{c} \qquad (8)$$

qo‘llaniladi. Bundan quyidagiga kelamiz:

$$ax+b=xt^2+2\sqrt{c}t.$$

Bundan, $x=\dfrac{2\sqrt{c}t-b}{a-t^2},\ \sqrt{ax^2+bx+c}=\dfrac{\sqrt{c}t^2-bt+a\sqrt{c}}{a-t^2},\ dx=2\cdot\dfrac{\sqrt{c}t^2-bt+\sqrt{c}a}{\left(a-t^2\right)^2}dt.$

Buni (7) ga qo‘yib, integral ostidagi ifodani ratsionallashtiramiz, uni integrallab, eski o‘zgaruvchiga qaytamiz:

$$t=\frac{\sqrt{ax^2+bx+c}-\sqrt{c}}{x}.$$

Agar $ax^2+bx+c$ kvadrat uchhad $\alpha$ va $\beta$ ildizlarga ega bo‘lsa, *uchinchi almashtirish* qo‘llaniladi. Bunday holda uchhad chiziqli ko‘paytiruvchilarga ajraladi:

$$ax^2+bx+c=a\left(x-\alpha\right)\left(x-\beta\right).$$

Agar $\sqrt{ax^2+bx+c}=t\left(x-\alpha\right)$ deb qaralsa,

$$a\left(x-\beta\right)=t^2\left(x-\alpha\right)$$

hosil bo‘ladi. Demak,

$$x=\frac{-a\beta+\alpha t^2}{t^2-a},\ \sqrt{ax^2+bx+c}=\frac{a\left(\alpha-\beta\right)t}{t^2-a},\ dx=\frac{2a\left(\beta-\alpha\right)t}{\left(t^2-a\right)^2}dt.$$

Hosil bo‘lgan ifodani integrallash qiyinchilik tug‘dirmaydi.

***1-misol.*** $\int\frac{dx}{\sqrt{a^2-x^2}}$ integralni Eylerning uchinchi almashtirishini qo‘llab, hisoblang.

***Yechilishi*:** Almashtirish olamiz:

$\sqrt{a^2-x^2}=t\left(a-x\right),\ x=a\cdot\frac{t^2-1}{t^2+1},\ dx=\frac{4at}{\left(t^2+1\right)^2}dt,\ \sqrt{a^2-x^2}=\frac{2at}{t^2+1}.$ U holda,

$$\int\frac{dx}{\sqrt{a^2-x^2}}=2\int\frac{dt}{t^2+1}=2arctgt+c=2arctg\sqrt{\frac{a+x}{a-x}}+c.$$

Berilgan integralni $\sqrt{a^2-x^2}=xt-a$ almashtirish, ya’ni ikkinchi almashtirish yordamida ham hisoblash mumkin:

$$\int\frac{dx}{\sqrt{a^2-x^2}}=-2\int\frac{dt}{t^2+1}=-2arctgt+c=-2arctg\frac{a+\sqrt{a^2-x^2}}{x}+c.$$

***2-misol.*** $\int\frac{dx}{x+\sqrt{x^2-x+1}}$ integralni Eyler almashtirishlari yordamida hisoblang.

***Yechilishi*:** Eylerning birinchi almashtirishini qo‘llaylik:

$$\sqrt{x^2-x+1}=t-x_3\quad x=\frac{t^2-1}{2t-1},\ dx=2\frac{t^2-t+1}{\left(2t-1\right)^2}dt.$$

U holda,

$$\int\frac{dx}{x+\sqrt{x^2-x+1}}=\int\frac{2t^2-2t+2}{t\left(2t-1\right)^2}dt=\int\left(\frac{2}{t}-\frac{3}{2t-1}+\frac{3}{\left(2t-1\right)^2}\right)dt=$$

$$=-\frac{3}{2}\cdot\frac{1}{2t-1}+2\ln\left|t\right|-\frac{3}{2}\left|2t-1\right|+c.$$

Berilgan ifodada quyidagi almashtirishni

$$t = x + \sqrt{x^2 - x + 1}$$

amalga oshirsak, u holda

$$\int \frac{dx}{x + \sqrt{x^2 - x + 1}} = -\frac{3}{2} \cdot \frac{1}{2x + 2\sqrt{x^2 - x + 1}} - \frac{3}{2}\ln\left|2x + 2\sqrt{x^2 - x + 1} - 1\right| +$$
$$+\sqrt{x^2 - x + 1} + c$$

hosil bo‘ladi. Endi, Eylerning uchinchi almashtirishini yuqoridagi integralni hisoblashda qo‘llaymiz:

$$\sqrt{x^2 - x + 1} = tx - 1,\ \ x = \frac{2t - 1}{t^2 - 1} - dx = -2 \cdot \frac{t^2 - t + 1}{\left(t^2 - 1\right)^2} dt,\ \ \sqrt{x^2 - x + 1} = \frac{t^2 - t + 1}{t^2 - 1},$$

$$x + \sqrt{x^2 - x + 1} = \frac{t}{t - 1}.$$

Integralni hisoblaymiz:

$$\int \frac{dx}{x + \sqrt{x^2 - x + 1}} = \int \frac{-2t^2 + 2t - 2}{t(t-1)(t+1)^2} dt = \int \left(\frac{3}{t} - \frac{1}{2} \cdot \frac{1}{t-1} - \frac{3}{2} \cdot \frac{1}{t+1} - \frac{3}{(t+1)^2}\right) dt =$$
$$= \frac{3}{t+1} + 2\ln|t| - \frac{1}{2}\ln|t-1| - \frac{3}{2}\ln|t+1| + c.$$

Hosil bo‘lgan natijaga $t = \dfrac{\sqrt{x^2 - x + 1} + 1}{x}$ ni qo‘yib, soddalashtiramiz:

$$\int \frac{dx}{x + \sqrt{x^2 - x + 1}} = \frac{3x}{\sqrt{x^2 - x + 1} + x + 1} + 2\ln\left|\sqrt{x^2 - x + 1} + 1\right| - \frac{1}{2}\ln\left|\sqrt{x^2 - x + 1} - x + 1\right| -$$
$$-\frac{3}{2}\ln\left|\sqrt{x^2 - x + 1} + x + 1\right| + c^1.$$

$c^1 = c + \dfrac{3}{2}$ deb olinsa, natija oldingi almashtirishlardagiga aynan teng bo‘ladi.

## MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi integrallarni hisoblang:

1. $\int \frac{dx}{1+\sqrt{x}}$.

2. $\int \frac{dx}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}$.

3. $\int \frac{1+\sqrt[6]{x}}{\sqrt[3]{x}+\sqrt{x}}dx$.

4. $\int \frac{1}{x}\sqrt{\frac{x-2}{x}}dx$.

5. $\int \frac{dx}{\left(1+\sqrt[4]{x}\right)^2\cdot\sqrt{x}}$.

6. $\int \frac{dx}{\sqrt{x}\left(1+\sqrt[4]{x}\right)^3}$.

7. $\int \frac{dx}{\sqrt{2x+1}+1}$.

8. $\int \frac{dx}{(1-x)\sqrt{1-x^2}}$.

9. $\int (x-2)\sqrt{\frac{1+x}{1-x}}dx$.

10. $\int \frac{dx}{x\left(1+2\sqrt{x}+\sqrt[3]{x}\right)}$.

11. $\int \frac{1}{x^3\sqrt{x^2+1}}dx$.

12. $\int \frac{x^3}{\sqrt{1+2x-x^2}}dx$.

13. $\int \frac{x^2}{\sqrt{9-x^2}}dx$.

14. $\int \frac{x^3}{\sqrt{4-x^2}}dx$.

15. $\int \frac{dx}{x\sqrt{a^2+x^2}}$.

## TRIGONOMETRIK FUNKSIYALARNI INTEGRALLASH

$\int sin^n x\, cos^m xdx$ ko‘rinishdagi integrallar quyidagicha topiladi:

a) Agar *n>0* toq bo’lsa, $cos\, x = t$, $sin\, xdx = -dt$ o‘rniga qo‘yish integralni ratsionallashtiradi.

**Misol 17.** $\int \sin^3 x\cdot\cos^2 xdx$ integralni toping.

**Yechilishi:** $\sin^3 x$ da bitta $\sin x$ ko‘paytuvchini ajratamiz va uni differensial ostiga kiritamiz:

$$\int sin^3 x\cdot cos^2 xdx = \int sin^2 x\cdot cos^2 x\, sin\, xdx = -\int sin^2 x\, cos^2 xd(cos\, x) = -\int (1-cos^2 x)cos^2 xd(cos\, x) =$$

$$= -\int (cos^2 x - cos^4 x)d(cos\, x) = C - \frac{1}{3}cos^3 x + \frac{1}{5}cos^5 x.$$

b) Agar *m>0* toq bo‘lsa, u holda $\sin x = t$, $\cos xdx = dt$ o’rniga qo‘yish integralni ratsionallashtiradi.

**Misol 18.** $\int \frac{\cos^3 xdx}{\sqrt[3]{\sin^4 x}}$ integralni toping.

**Yechilishi:**

$$\int\frac{\cos^3 xdx}{\sin^{\frac{4}{3}} x}=\int\frac{\cos^2 x\cos xdx}{\sin^{\frac{3}{4}} x}=\int\frac{(1-\sin^2 x)d(\sin x)}{\sin^{\frac{4}{3}} x}=\int(\sin^{-\frac{4}{3}} x-\sin^{\frac{2}{3}} x)d(\sin x)=$$

$$=-3\sin^{-\frac{1}{3}} x-\frac{3}{5}\sin^{\frac{5}{3}} x+C=C-\frac{3}{\sqrt[3]{\sin x}}-\frac{3}{5}\sqrt[3]{\sin^5 x}.$$

**Misol 19.** $\int\sin^4 xdx$ integralni toping.

**Yechilishi:**

$$\int\sin^4 xdx=\int(\sin^2 x)^2dx=\int\left(\frac{1-\cos 2x}{2}\right)^2dx=\frac{1}{4}(1-2\cos 2x+\cos^2 2x)dx=$$

$$=\frac{1}{4}(x-\sin 2x+\int\cos^2 2xdx)=\frac{1}{4}(x-\sin 2x+\frac{1}{2}\int(1+\cos 4x)dx)=$$

$$=\frac{1}{4}(x-\sin 2x+\frac{1}{2}x+\frac{1}{8}\sin 4x)+C=\frac{1}{4}\left(\frac{3x}{2}-\sin 2x+\frac{1}{8}\sin 4x\right)+C.$$

**Misol 20.** $\int\cos x*\cos 2x*\cos 4xdx$ integralni toping.

**Yechilishi.** Ko‘paytmadan yig‘indiga o‘tish formulasi ikki marta qo‘llaniladi:

$$\int\cos x*\cos 2x*\cos 4xdx=\frac{1}{2}\int[\cos 3x+\cos(-x)]\cos 4xdx=\frac{1}{2}\int\cos 3x\cos 4xdx+\frac{1}{2}\int\cos x\cos 4xdx=$$

$$=\frac{1}{4}\int[\cos 7x+\cos(-x)]dx+\frac{1}{4}\int[\cos 5x+\cos(-3x)]dx=\frac{1}{4}(\frac{1}{7}\sin 7x+\frac{1}{5}\sin 5x+\frac{1}{3}\sin 3x+\sin x)+c.$$

## MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

### Quyidagi integrallarni toping:

1. $\int\sin^4 xdx.$
2. $\int\cos^3 xdx.$
3. $\int\frac{dx}{\sin 2x}.$
4. $\int\frac{dx}{\cos 2x}.$
5. $\int\frac{dx}{\sin x\cdot\cos^4 x}.$

12. $\int\frac{tgx}{tgx-3}dx.$
13. $\int\frac{dx}{\sin^4 x+\cos^4 x}.$
14. $\int\frac{\sin^2 x}{\sin x+2\cos x}dx.$
15. $\int\frac{1}{1+\sin x}dx.$
16. $\int\cos^4 xdx.$

6. $\int tg^3 x dx.$

7. $\int \sin 2x \cdot \cos 4x dx.$

8. $\int \cos x \cdot \cos 4x dx.$

9. $\int \frac{\cos^2 x}{\sin 4x} dx.$

10. $\int \frac{\sin x + \sin^3 x}{\cos 2x} dx.$

11. $\int \frac{\sin 2x}{3 + 4\sin^2 x} dx.$

17. $\int \sin^3 x \cos^3 x dx.$

18. $\int \sin 3x \cdot \sin 4x dx.$

19. $\int \sqrt{4 - x^2} dx.$

20. $\int \frac{x^2 dx}{\sqrt{9 - x^2}}.$

21. $\int x\sqrt{a - x} dx.$

## INTEGRALLASH FORMULALARI

**c – o‘zgarmas son**

**Integrallash qoidalari:**

$I. \int f(x)dx = F(x) + c$

$II. \int kf(x)dx = k\int f(x)dx + c. \ k - const., k \neq 0$

$III. \int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx + c$

$IV. \int dF(x) = F(x) + c., \ d[\int f(x)dx] = f(x)dx.$

## INTEGRALLAR JADVALI VA BOSHQA MUHIM FORMULALAR:

1. $\int dx = x + c.$

2. $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1.$

3. $\int \frac{dx}{x} = ln\,|\,x\,| + c.$

4. $\int a^x dx = \frac{a^x}{ln\,a} + c.$

5. $\int e^x dx = e^x + c.$

6. $\int sin\,xdx = -\,cos\,x + c.$

7. $\int cos\,xdx = sin\,x + c.$

8. $\int \frac{dx}{cos^2 x} = tgx + c.$

9. $\int \frac{dx}{sin^2 x} = -ctgx + c.$

10. $\int \frac{dx}{\sqrt{1-x^2}} = arcsin\,x + c.$

11. $\int \frac{dx}{1+x^2} = arctgx + c.$

12. $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + c.$

$$13.\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \ (a < c < b).$$

$$14.\ \int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a).$$

$$15.\ \int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\varphi'(t)dt. \qquad 16.\ \int_a^b udv = uv\Big|_a^b - \int_a^b vdu.$$

$$17.\ V = \pi\int_a^b f^2(x)dx.$$

## NYUTON – LEYBNIS FORMULASI

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a).$$

**Misol 21.**

$$\int_{-1}^{2}(x-3)^2 dx = \int_{-1}^{2}(x^2 - 6x + 9)dx = \int_{-1}^{2} x^2 dx - \int_{-1}^{2} 6xdx + \int_{-1}^{2} 9dx =$$

$$= \int_{-1}^{2} x^2 dx - 6\int_{-1}^{2} x dx + 9\int_{-1}^{2} dx = \frac{x^{2+1}}{2+1}\Big|_{-1}^{2} - 6\frac{x^{1+1}}{1+1}\Big|_{-1}^{2} + 9x\Big|_{-1}^{2} =$$

$$= \frac{x^3}{3}\Big|_{-1}^{2} - 6\frac{x^2}{2}\Big|_{-1}^{2} + 9x\Big|_{-1}^{2} = \frac{1}{3} * x^3\Big|_{-1}^{2} - 3x^2\Big|_{-1}^{2} + 9x\Big|_{-1}^{2} =$$

$$= \frac{1}{3}[2^3 - (-1)^3] - 3[2^2 - (-1)^2] + 9[2 - (-1)] = \frac{1}{3}(8+1) - 3(4-1) + 9(2+1) =$$

$$= \frac{1}{3} * 9 - 3 * 3 + 9 * 3 = 3 - 9 + 27 = 21.$$

**Misol 22.**

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin 6x dx = \frac{1}{6}(-\cos 6x)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = -\frac{1}{6}\cos 6x\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = -\frac{1}{6}[\cos 6 * \frac{\pi}{2} - \cos 6 * \frac{\pi}{3}] =$$

$$= -\frac{1}{6}[\cos 3\pi - \cos 2\pi] = -\frac{1}{6}[-1-1] = \frac{1}{3}.$$

## O'ZGARIVCHILARNI ALMASHTIRISH

$$\int_{a}^{b} f(x)dx = \int_{\alpha}^{\beta} f(\varphi(t))\varphi'(t)dt$$

**Misol 23.**

$$\int_{0}^{1} \frac{x^2 dx}{(x+1)^2} = \left| \begin{array}{l} 1. O'zgaruvchi\ almashtiriladi: \\ x+1 = t \Leftrightarrow x = t-1 \Leftrightarrow \\ \Leftrightarrow dx = dt; \\ 2. Chegaralar\ almashtiriladi: \\ x = 0 \Rightarrow t = 1;\ x = 1 \Rightarrow t = 2. \end{array} \right| = \int_{1}^{2} \frac{(t-1)^2}{t^2} dt = \int_{1}^{2} \frac{t^2 - 2t + 1}{t^2} dt =$$

$$= \int_{1}^{2} (1 - \frac{2}{t} + \frac{1}{t^2}) dt = \int_{1}^{2} dt - 2\int_{1}^{2} \frac{dt}{t} + \int_{1}^{2} t^{-2} dt = t\Big|_{1}^{2} - 2\ln|t|\Big|_{1}^{2} + \frac{t^{-2+1}}{-2+1}\Big|_{1}^{2} = t\Big|_{1}^{2} - 2\ln|t|\Big|_{1}^{2} - \frac{1}{t}\Big|_{1}^{2} =$$

$$= 2 - 1 - 2[\ln|2| - \ln|1|] - [\frac{1}{2} - \frac{1}{1}] = 1 - 2[\ln 2 - 0] - \frac{1}{2} + 1 = 2 - \frac{1}{2} - 2\ln 2 = \frac{3}{2} - 2\ln 2.$$

**Misol 24.**

$$\int_0^{\frac{\pi}{2}} \cos^2 x \sin x dx = \left| \begin{array}{l} 1.\ t = \cos x; \\ dt = -\sin x dx; \\ \sin x dx = -dt; \\ 2. x = 0 \Rightarrow t = 1; \\ x = \frac{\pi}{2} \Rightarrow t = 0. \end{array} \right| = \int_1^0 t^2(-dt) = -\int_1^0 t^2 dt = \int_0^1 t^2 dt = \frac{t^3}{3}\Big|_0^1 = \frac{1}{3}t^3\Big|_0^1 =$$

$$= \frac{1}{3}[1^3 - 0^3] = \frac{1}{3}.$$

## BO‘LAKLAB INTEGRALLASH

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu.$$

**Misol 25.**

$$\int_0^{\pi} x \sin x dx = \left| \begin{array}{l} u = x \Leftrightarrow du = dx; \\ dv = \sin x dx \Leftrightarrow \\ \Leftrightarrow \int dv = \int \sin x dx \Rightarrow \\ \Rightarrow v = -\cos x. \end{array} \right| = -x * \cos x\Big|_0^{\pi} + \int_0^{\pi} \cos x dx = -x * \cos x\Big|_0^{\pi} + \sin x\Big|_0^{\pi} =$$

$$= -[\pi * \cos \pi - 0 * \cos 0] + \sin \pi - \sin 0 = -(\pi * (-1) - 0) + 0 - 0 = \pi.$$

## MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

### Quyidagi aniq integrallarni hisoblang:

1. $\int_0^3 x^2 dx$; **Javobi: 9.**
2. $\int_1^2 \frac{1}{x^3} dx$ **Javobi:** $\frac{3}{8}$.
3. $\int_{-2}^3 2x dx$ **Javobi:5.**
4. $\int_{-2}^{-1} (5 - 4x) dx$; **Javobi: 11.**
5. $\int_{-1}^1 (x^2 + 1) dx$; **Javobi:** $\frac{2}{3}$.
6. $\int_0^{\frac{\pi}{2}} \sin(2x + \frac{\pi}{3}) dx$ ; **Javobi: 0,5.**

**7.** $\int_4^9 \frac{1}{\sqrt{x}}dx$ **Javobi: 2.**

**8.** $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos xdx$ **;** **Javobi: 2.**

**Quyidagi aniq integrallarni o‘zgarivchini almashtirish orqali hisoblang:**

**9.** $\int_0^1 \frac{x^3dx}{(x-1)^2}$, $(x-1=t);$ **Javobi: -1,5.**

**12.** $\int_0^5 x\sqrt{x+4}dx,\ (t=\sqrt{x+4});$ **Javobi:** $33\frac{11}{15}$.

**10.** $\int_4^9 \frac{dx}{\sqrt{x}-1}$, $(\sqrt{x}=t);$ **Javobi:** $2(1+\ln 2)$.

**13.** $\int_0^1 x\sqrt{1+x^2}dx,\ (t=1+x^2);$ **Javobi:** $\frac{2\sqrt{2}-1}{3}$.

**11.** $\int_3^8 \frac{x}{\sqrt{1+x}}dx,\ (\sqrt{1+x}=t);$ **Javobi:** $\frac{32}{3}$.

**14.** $\int_0^{\frac{\pi}{4}} \frac{dx}{2\cos x+3},\ (t=tg\frac{x}{2});$ **Javobi:** $\frac{2}{\sqrt{5}}arctg\frac{1}{\sqrt{5}}$.

**Quyidagi aniq integrallarni bo‘laklab integrallash orqali hisoblang:**

**15.** $\int_0^e xe^x dx,\ (u=x);$ **Javobi:** $e^e(e-1)+1.$

**18.** $\int_0^1 arctgxdx,\ (u=arctgx);$ **Javobi:** $\frac{\pi}{4}-\ln\sqrt{2}.$

**16.** $\int_0^{\pi} x^2\ln xdx,\ (u=\ln x);$ **Javobi: 2.**

**19.** $\int_0^{\pi} x\cos xdx,\ (u=x);$ **Javobi: 2.**

**17.** $\int_0^{\frac{\pi}{2}} x\cos xdx,\ (u=x);$ **Javobi: -2.**

**20.** $\int_1^8 x\sin xdx,\ (u=x);$ **Javobi:** $\pi$.

## ANIQ INTEGRALNING TATBIQI

### Aniq integral yordamida yuzni hisoblash

1. $f(x)\geq 0, x=a, x=b, \ (0x) \ bo'lsa \ , \quad S=\int\limits_a^b f(x)dx.$

2. $f(x)\leq 0, x=a, x=b, \ (0x) \ bo'lsa \ , \quad S=\left|\int\limits_a^b f(x)dx\right|=-\int\limits_a^b f(x)dx.$

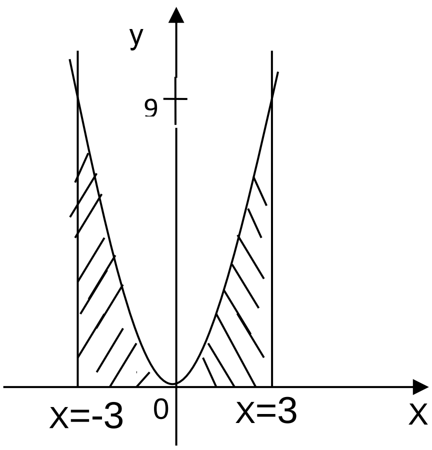
3. $f(x)\geq g(x), x=a, x=b, \quad bo'lsa \ , \quad S=\int\limits_a^b [f(x)-g(x)]dx.$

**Misol 26.** $y=x^2$ parabola, $\boldsymbol{x=-3}$ va $\boldsymbol{x=3}$ to‘g‘ri chiziqlar, **(O*x*)** abssissa o‘qi bilan chegaralangan yuzni hisoblang.

**Yechilishi.**

| | | | | | | |
|---|---|---|---|---|---|---|
| x | -3 | -2 | -1 | 0 | 1 | 2 |
| $y=x^2$ | 9 | 4 | 1 | 0 | 1 | 4 |

**1- usul.**




$$S=\int\limits_a^b f(x)dx=\int\limits_{-3}^{3} x^2dx=\frac{1}{3}x^3\Big|_{-3}^{3}=\frac{1}{3}[3^3-(-3)^3]=$$

$$=\frac{1}{3}(27+27)=\frac{54}{3}=18 \quad kv.\ birlik.$$

**2- usul.**

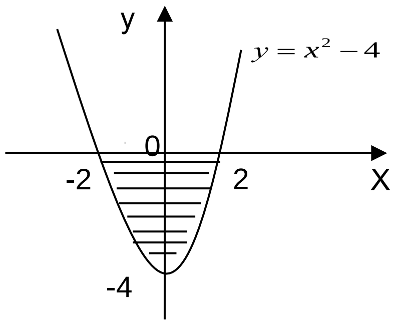
$$S=2\int\limits_0^3 x^2dx=2*\frac{1}{3}x^3\Big|_0^3=\frac{2}{3}[3^3-0^3]=$$

$$=\frac{54}{3}=18 \ kv.\ birlik.$$

**Misol 27.** $\boldsymbol{y=x^2-4}$ parabola va **(*0x*)** o‘qi bilan chegaralangan yuzni hisoblang.

**Yechilishi.**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *x* | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| $y=x^2-4$ | 5 | 0 | -3 | -4 | 3 | 0 | 5 |




Intervalning ***a*** va ***b*** chegaralarini topish uchun ***y=x²-4*** funksiyani, ***(0x)*** o'qining tenglamasi ***y = 0*** bilan birgalikda sistema qilib yechish kerak.

$$\begin{cases} y = x^2 - 4 \\ y = 0 \end{cases} \Rightarrow x^2 - 4 = 0 \Rightarrow x^2 = 4 \Rightarrow \sqrt{x^2} = \sqrt{4} \Rightarrow |x| = 2 \Rightarrow$$

$$\Rightarrow \pm x = 2 \Rightarrow x = \pm 2 \Rightarrow \begin{cases} x_1 = -2 \\ x_2 = 2 \end{cases} \Rightarrow \begin{cases} a = -2; \\ b = 2. \end{cases}$$

**1-usul**

$$S = \left| \int_a^b f(x)dx \right| = \left| \int_{-2}^{2} (x^2 - 4)dx \right| = \left| \int_{-2}^{2} x^2 dx - 4\int_{-2}^{2} dx \right| = \left| \frac{1}{3} x^3 \Big|_{-2}^{2} - 4x\Big|_{-2}^{2} \right| =$$

$$= \left| \frac{1}{3} [2^3 - (-2)^3] - 4[2 - (-2)] \right| =$$

$$= \left| \frac{16}{3} - 16 \right| = \left| \frac{16 - 3 * 16}{3} \right| = \left| -\frac{32}{3} \right| = \frac{32}{3} = 10\frac{2}{3} \; kv.\ birlik.$$

**2-usul.**

$$S = -2\int_0^2 (x^2 - 4)dx = -2\int_0^2 x^2 dx + 8\int_0^2 dx = -2 * \frac{x^3}{3}\Bigg|_0^2 + 8x\Big|_0^2 = -\frac{2}{3}[2^3 + 0^3] + 8[2 - 0] =$$

$$= -\frac{16}{3} + 16 = \frac{-16 + 48}{3} = \frac{32}{3} = 10\frac{2}{3} \; kv.birlik$$

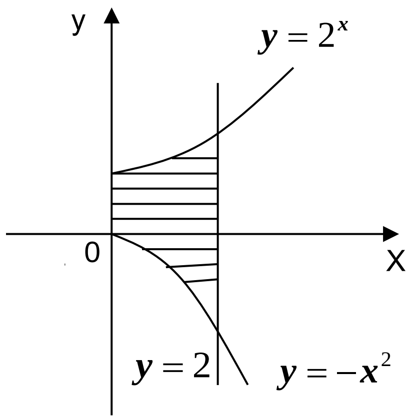
**Misol 28.** $y = x^2 - 2x$ egri chiziq, ***x = - 1, x = 1*** to'g'ri chiziqlar va (***0x***) o'qi bilan chegaralangan shakilning yuzini hisoblang.

**Yechilishi.**

$$S = \int_{-1}^{0} (x^2 - 2x)dx - \int_0^1 (x^2 - 2x)dx = \left( \frac{x^3}{3} - x^2 \right)\Bigg|_{-1}^{0} - \left( \frac{x^3}{3} - x^2 \right)\Bigg|_0^1 = (\frac{1}{3} + 1) - (\frac{1}{3} - 1) = \frac{1}{3} + 1 - \frac{1}{3} + 1 = 2.$$

**Misol 29.** $f(x) = 2^x$ ***va*** $g(x) = -x^2$ funksiyalar, **(0*x*)** o'qi va ***x=2*** to'g'ri chiziq bilan chegaralangan yuzni hisoblang.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *x* | 0 | 1 | 2 | *x* | 0 | 1 | 2 |
| *f(x)=2ˣ* | 1 | 2 | 4 | *g(x)=-x²* | 0 | -1 | -4 |




$$S = \int_a^b [f(x) - g(x)]dx = \int_0^2 [2^x - (-x^2)]dx = \int_0^2 2^x dx + \int_0^2 x^2 dx =$$

$$\frac{2^x}{\ln 2}\Bigg|_0^2 + \frac{1}{3}x^3\Bigg|_0^2 = \frac{2^2}{\ln 2} - \frac{2^0}{\ln 2} + \frac{8}{3} = \frac{4-1}{\ln 2} + \frac{8}{3} = \frac{3}{\ln 2} + \frac{8}{3} = \frac{9 + 8\ln 2}{3\ln 2} =$$

$$= \frac{9 + \ln 2^8}{\ln 2^3} = \frac{9 + \ln 256}{\ln 8} \; kv.\, birlik.$$

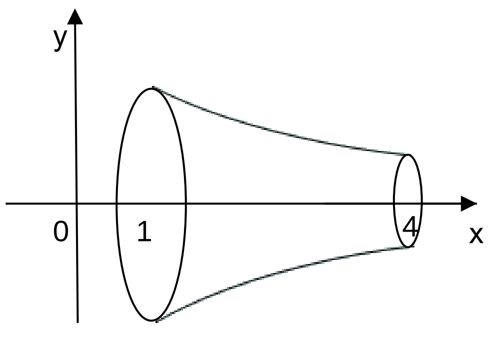
## Aniq integral yordamida hajmni hisoblash

$$V = \pi \int_a^b f^2(x)dx$$

**Misol 30.** ***xy=4, x=1, x=4*** chiziqlar bilan chegaralangan figuraning **(*0x*)** o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini hisoblang.

**Yechilishi.**

| $x$ | 1 | 2 | 4 |
|---|---|---|---|
| $y = \frac{4}{x}$ | 4 | 2 | 1 |




$$V = \pi \int_1^4 \left(\frac{4}{x}\right)^2 dx = \pi \int_1^4 \frac{16}{x^2} dx = 16\pi \int_1^4 x^{-2} dx = 16\pi \frac{x^{-2+1}}{-2+1}\Bigg|_1^4 = -16\pi\, x^{-1}\Big|_1^4 = -16\pi * \frac{1}{x}\Bigg|_1^4 =$$

$$= -16\pi\left[\frac{1}{4} - 1\right] = -16\pi * \frac{1-4}{4} = 12\pi \; kub\, birlik.$$

**Misol 31.** $y^2+x-4=0$ **va** $x$**=0** chiziqlar bilan chegaralangan figuraning ($0x$) o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

**Yechilishi.**

$$\begin{cases} x=4-y^2 \\ x=0 \end{cases} \Rightarrow 4-y^2=0 \Rightarrow \begin{cases} y_1=-2 \\ y_2=2 \end{cases} \Rightarrow \begin{cases} a=-2; \\ b=2. \end{cases}$$

$$V=\pi\int_{-2}^{2}(4-y^2)^2dy=2\pi\int_{0}^{2}(16-8y^2+y^4)dy=2\pi(16y-\frac{8}{3}y^3+\frac{1}{5}y^5)\Big|_0^2=$$

$$=2\pi(16*2-\frac{8}{3}2^3+\frac{1}{5}2^5)=2\pi(32-\frac{64}{3}+\frac{32}{5})=2\pi\frac{480-320+96}{15}=$$

$$=2\pi\frac{265}{15}=\frac{512\pi}{15}=34\frac{2}{15}\pi \text{ kub birlik.}$$

## MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

### Qo'yidagi chiziqlar bilan chegaralangan figuraning yuzini hisoblang:

1. $y=x^2,\ y=0, x=0,\ x=2;$ Javobi: $2\frac{2}{3}$. kv. birlik.

2. $y=x^2,\ y=0,\ x=-2;$ Javobi: $2\frac{2}{3}$. kv. birlik.

3. $y=x^3,\ y=0,\ x=2;$ Javobi: 4. kv. birlik.

4. $y=\sqrt{x},\ y=0,\ x=4;$ Javobi: $5\frac{1}{3}$. kv. birlik.

5. $y=\frac{1}{\sqrt{x}},\ y=0,\ x=1, x=4;$ Javobi: 2. kv. birlik.

6. $y=\frac{3}{\sqrt{x}},\ y=0,\ x=1; x=4;$ Javobi: 6. kv. birlik.

7. $y=x^2,\ y=2x.$ Javobi: $1\frac{1}{3}$. kv. birlik.

### Quyidagi chiziqlar bilan chegaralangan figuraning ko'rsatilgan kordinata o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini hisoblang.

8. $y=x-1,\ x=-3, x=0,\ y=0;\ (0x)$. **Javobi:** $21\pi$ *kub birlik*.

9. $y = x + 2,\ x = -3, x = 0,\ y = 0;\ (0x)$. **Javobi:** $3\pi$ ***kub birlik.***

10. $y = x - 1,\ x = -1, x = 2,\ y = 0;\ (0x)$. **Javobi:** $3\pi$ ***kub birlik.***

11. $y = x^3,\ \ x = 4,\ (0x)$. **Javobi:** $64\pi$ ***kub birlik.***

12. $y = 2x - x^2,\ \ y = 0,\ (0x)$. **Javobi:** $18{,}9\pi$ ***kub birlik.***

13. $y = |x + 1|,\ x = -3,\ x = 0,\ y = 0;\ (0x)$. **Javobi:** $1\frac{1}{3}\pi$ ***kub birlik.***

14. $x \cdot y = 9,\ \ y = 3,\ \ y = 10,\ \ (0y)$. **Javobi:** $3\pi$ ***kub birlik.***

15. $y = |x + 2|,\ x = -3,\ x = 0,\ y = 0;\ (0x)$. **Javobi:** $3\pi$ ***kub birlik.***

16. $y = |x - 1|,\ x = -1,\ x = 2,\ y = 0;\ (0x)$. **Javobi:** $3\pi$ ***kub birlik.***

## Aniq integrallarni taqribiy hisoblash

### To‘g‘ri to‘rtburchaklar formulasi

To‘g‘ri burchakli koordinatalar sistemasida $y = f(x)$ uzluksiz funksiya berilgan bo’lsin. Integrallash oralig’i $[a, b]$ ni $x_0, x_1, x_2, x_3, \ldots, x_n$ nuqtalar yordamida $n$ ta teng bo’laklarga ajratamiz. U holda, bir bo’lakning uzunligi

$$h = \frac{x_n - x_0}{n} \quad \text{yoki} \quad h = \frac{b - a}{n} \qquad (1)$$

dan iborat bo’ladi. $y = f(x)$, $y = 0$, $x = a,\ x = b$ chiziqlar bilan chegaralangan egri chiziqli figura ham $n$ ta bo’lakka ajralib, ordinatalari $y_0, y_1, y_2, \ldots, y_n$ bo’ladi. Bo’laklar kami va ortig’i bilan to’g’ri to’rtburchaklarga to’ldiriladi.

Kami bilan to’ldirilgan to’g’ri to’rtburchaklarning yuzalari yig’indisi:

$$S = \int_a^b y dx \approx \frac{b - a}{n}(y_0 + y_1 + y_2 + \ldots + y_{n-1}). \qquad (2)$$

Ortig’i bilan to’ldirilgan yuzalar yig’indisi esa

$$S = \int_a^b y dx \approx \frac{b - a}{n}(y_1 + y_2 + \ldots, + y_n). \qquad (3)$$

dan iborat bo’ladi.

Agar yuzalar yana ikkiga bo'linsa, (2) va (3) larning xatosi kamaya boradi. $n \to \infty$ va $h \to 0$ bo'lsa, ular aniq integralning yanada haqiqiyroq qiymatini beradi. (3) ni quyidagicha ifodalasa ham bo'ladi:

$$S = \int_a^b y dx \approx \frac{b-a}{n}\left( y_{\frac{1}{2}} + y_{\frac{3}{2}} + \ldots + y_{\frac{2n-1}{2}} \right). \qquad (3')$$

## Trapetsiyalar formulasi

Trapetsiyalar formulasini keltirib chiqarishda $[a, b]$ kesmani $n$ ta teng bo'laklarga ajratamiz. U holda, bo'linish nuqtalari (ya'ni absissalar)

$x_0 = a, x_1, x_2, \ldots, x_n = b$ bo'ladi. Kesmalar o'zaro teng bo'lganligi uchun ularni umumiy holda $h$ bilan belgilasak,

$$x_0 = a, \;\; x_1 = x_0 + h, \;\; x_2 = x_1 + h, \ldots, x_n = x_{n-1} + h = b \qquad (4)$$

bo'ladi. Ordinata chiziqlari mos ravishda

$$y_0, y_1, y_2, \ldots, y_n \qquad (5)$$

lardan iborat bo'lib, ular ketma- ket tutashtirilsa, siniq chiziqlar hosil bo'ladi. Absissa, ordinatalar va funktsiya chiziqlari bilan chegaralangan egri chiziqli trapetsiyalar hosil bo'lib, ularning yuzalari

$$S_1 = \frac{y_0 + y_1}{2} h, \; S_2 = \frac{y_1 + y_2}{2} h, \ldots, S_n = \frac{y_{n-1} + y_n}{2} h \qquad (6)$$

lardan iborat. U holda, egri chiziqli trapetsiyasimon $S$ figuraning yuzi quyidagicha bo'ladi:

$$S = \int_a^b y dx = S_1 + S_2 + \ldots + S_n = \frac{y_0 + y_1}{2} h + \frac{y_1 + y_2}{2} h + \ldots + \frac{y_{n-1} + y_n}{2} h =$$

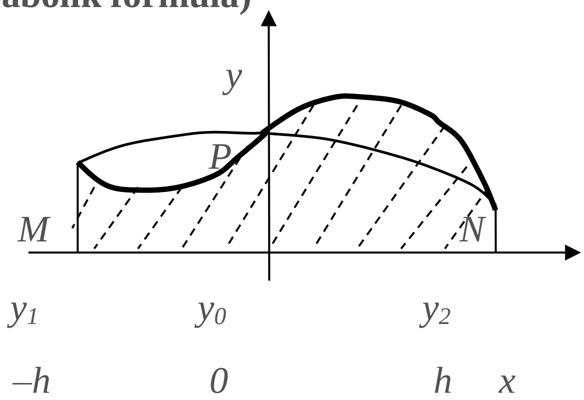
$$= \frac{h}{2}\left( \frac{y_0 + y_n}{2} + y_1 + y_2 + \ldots + y_{n-1} \right). \qquad (7)$$

Bunda $h = \dfrac{b-a}{n}$ dan ibort.

Bu formulaga ***trapetsiyalar formulasi*** deyiladi. (6) formulaning xatosi to'g'ri to'rtburchaklar formulasiga nisbatan kichikroq. Agar $n \to \infty$ bo'lsa, $h \to 0$ bo'lib, hato yanada kamayadi. Natijada, bu formula integralning izlangan qiymatini yetarlicha aniqlikda beradi.

**Simpson formulasi (Parabolik formula)**

Yuqoridan $y = f(x)$ egri chiziqli funktsiya, yon tomonlari $x = -h$ va $x = h$ kesmalar, pastdan $y = 0$ absissalar o'qi bilan chegaralangan figuraning yuzini topish talab qilinsin.




Agar $h$ cheksiz kichik bo'lsa, $y = f(x)$ funktsiyani taqriban $M(-h;\, y_1)$, $P(0;\, y_0)$ va $N(h;\, y_2)$ nuqtalardan o'tuvchi

$$Y = \alpha x^2 + \beta x + \lambda \qquad (7)$$

parabola yoyi bilan almashtirish mumkin bo'ladi. U holda, bu yoy bilan chegaralangan figuraning yuzi quyidagicha bo'ladi:

$$S = \int_{-h}^{h} \left(\alpha x^2 + \beta x + \lambda\right) dx = \left( \frac{\alpha x^3}{3} + \frac{\beta x^2}{2} + \lambda x \right)\Bigg|_{-h}^{h} = \frac{2\alpha}{3} h^3 + 2\lambda h. \qquad (8)$$

*MPN* yoy o'tgan nuqtalarning $x = -h$, $x = 0$ va $x = h$ absissalarini (7) ga ketma – ket qo'yamiz:

$$y_1 = \alpha h^2 - \beta h + \lambda, \quad y_0 = \gamma\,, \; y_2 = \alpha h^2 + \beta h + \lambda.$$

Bulardan, $\gamma = y_0$ va $\alpha = \dfrac{y_1 - 2y_0 + y_2}{2h^2}.$ (9)

(9) qiymatlarini (8)ga qo'ysak, ***parabolik yoy formulasi*** kelib chiqadi:

$$\int_{-h}^{h} y\,dx = \frac{1}{3} h\left(y_1 - 2y_0 + y_2\right) + 2y_0 h = \frac{h}{3}\left(y_1 + 4y_0 + y_2\right). \qquad (10)$$

Bu formulaning egri chiziqli figura yuzini topishdagi xatosi «***to'g'ri burchakli to'rtburchaklar***» va «***trapetsiyalar formulasi***» ga nisbatan kichikroqdir.

Koordinatalar sistemasini parallel ko'chirishni hisobga olsak, (10) formulani quyidagicha ham ifodalash mumkin:

$$\int_a^b ydx = \frac{h}{3}\left( y(a) + 4y\left(\frac{a+b}{2}\right) + y(b)\right).$$

(11)

Agar $[a, b]$ kesma $n$ bo'lakka ajratilsa, undagi egri chiziqli yuzalar soni ham $n$ ta bo'ladi. U holda, ***parabolik yoki Simpson formulasi***ning umumiy ko'rinishi quyidagicha bo'ladi:

$$S = \int_a^b ydx = \frac{b-a}{3n}\left(\frac{y_0 + y_n}{2} + (y_1 + y_2 + \ldots + y_{n-1}) + 2\left(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \ldots + y_{n-\frac{1}{2}}\right)\right) \quad (12)$$

yoki

$$S = \int_a^b ydx = \frac{h}{6n}\left((y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1}) + 4\left(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \ldots + y_{n-\frac{1}{2}}\right)\right). \quad (13)$$

***Misol.*** Quyidagi integralning $n = 10$ bo'lganda taqribiy qiymatini toping: $\int_0^1 \frac{dx}{1+x^2}$. $\left(\approx \frac{\pi}{4} \approx 0{,}785398\ldots\right)$.

***Yechilishi:*** Berilgan integralning taqribiy qiymatini «*to'g'ri to'rtburchaklar formulasi*», «*trapesya formulasi*» va «*Simpson formulasi*» yordamida hisoblab ko'rib, natijalarini taqqoslab ko'ramiz. (Uning qiymati $\frac{\pi}{4} \approx 0{,}785398 \ldots$ dan iborat).

1) To'g'ri to'rtburchaklar formulasi yordamida taqribiy qiymatini topish uchun ($3'$) formuladan foydalanamiz. Buning uchun $[a, b]$ kesmani, ya'ni $[0, 1]$ ni 10 bo'lakka ajratamiz va quyidagilarni hosil qilamiz (bunda $k = 0{,}10 \quad \textit{в}a \quad \frac{b-a}{n} = \frac{1}{10}$ ):

$x_{\frac{1}{2}} = 0{,}55$ $\qquad$ $y_{\frac{1}{2}} = 0{,}9975$

$x_{\frac{3}{2}} = 0{,}15$ $\qquad$ $y_{\frac{3}{2}} = 0{,}9780$

$x_{\frac{5}{2}} = 0{,}25$ $\qquad$ $y_{\frac{5}{2}} = 0{,}9412$

$x_{\frac{7}{2}} = 0{,}35$ $\qquad$ $y_{\frac{7}{2}} = 0{,}8909$

$x_{\frac{9}{2}} = 0{,}45$ $\qquad$ $y_{\frac{9}{2}} = 0{,}8316$

$x_{\frac{11}{2}} = 0{,}55$ $\qquad$ $y_{\frac{11}{2}} = 0{,}7678$

$x_{\frac{13}{2}} = 0{,}63$ $\qquad$ $y_{\frac{13}{2}} = 0{,}7029$

$x_{\frac{15}{2}} = 0{,}75$ $\qquad$ $y_{\frac{15}{2}} = 0{,}6400$

$x_{\frac{17}{2}} = 0{,}85$ $\qquad$ $y_{\frac{17}{2}} = 0{,}5806$

$x_{\frac{19}{2}} = 0{,}95$ $\qquad$ $y_{\frac{19}{2}} = 0{,}5256.$

Yig'indisi:7,8561

U holda, ($3'$) dan:

$$\int_0^1 y dx = \int_0^1 \frac{dx}{1+x^2} = \frac{1}{10} \cdot 7{,}8561 = 0{,}78561 .$$

Demak, integral qiymati 0,78361 bo'lib , uning xatosi taxminan 0,0002 ga teng.

2) Trapetsiyalar formulasi yordamida yuqoridagi integralni hisoblayiz. $n = 10$ deb, to'rtta raqamgacha hisoblaymiz. Trapesya formulasiga asosan quyidagilarni aniqlaymiz:

$x_0 = 0{,}0$ $\qquad$ $y_0 = 1{,}0000$

$x_{10} = 1{,}0$ $\qquad$ $y_{10} = 0{,}5000$

Yig'indisi:1,5000

$x_1 = 0{,}1$ $\qquad$ $y_1 = 0{,}9901$

$x_2 = 0,2$ $\qquad$ $y_2 = 0,9615$

$x_3 = 0,3$ $\qquad$ $y_1 = 0,9174$

$x_4 = 0,4$ $\qquad$ $y_4 = 0,8621$

$x_5 = 0,5$ $\qquad$ $y_5 = 0,8000$

$x_6 = 0,6$ $\qquad$ $y_6 = 0,7353$

$x_7 = 0,7$ $\qquad$ $y_7 = 0,6711$

$x_8 = 0,8$ $\qquad$ $y_8 = 0,6098$

$x_9 = 0,9$ $\qquad$ $y_9 = 0,5525$

Yig'indisi: 7,0996

U holda, (6) formuladan:

$$\int_0^1 ydx = \int_0^1 \frac{dx}{1+x^2} = \frac{1}{10}\cdot\left(\frac{1,5000}{2} + 7,0998\right) = 0,78498\,.$$

Demak, trapetsiyalar formulasi qo'llanib, integral hisoblanganda, u 0,78998 dan iborat bo'ldi. Topilgan taqribiy qiymat haqiqiy qiymatdan 0,0004 ga farq qiladi.

3) Simpson formulasi yordamida berilgan integralning taqribiy qiymatini topamiz. Buning uchun $n = 2$ deb, beshta raqamgacha hisoblaymiz. Bunda $\frac{b-a}{3n} = \frac{1}{6}$ dan iborat bo'ladi. U holda,

$x_0 = 0$ $\qquad$ $\frac{1}{2}y_0 = 0,50000$

$x_{\frac{1}{2}} = 0,25$ $\qquad$ $2y_{\frac{1}{2}} = 1,88235$

$x_1 = 0,50$ $\qquad$ $y_1 = 0,80000$

$x_{\frac{3}{2}} = 0,75$ $\qquad$ $2y_{\frac{3}{2}} = 1,28000$

$x_2 = 1{,}00$ $\qquad\qquad \frac{1}{2} y_2 = 0{,}25000$

Yig'indi:4,71235

(12) formulaga asosan:

$$\int_0^1 y dx = \int_0^1 \frac{dx}{1+x^2} = \frac{1}{6} \cdot 4{,}71235 = 0{,}78539$$

ni hosil qilamiz. Topilgan taqribiy qiymat haqiqiy qiymatdan 0,00001 ga farq qiladi.

Demak, Simpson formulasi yordamida berilgan integralni topish yuqoridagi ( $3'$ ) va (6) formulalar bilan topishga nisbatan ancha aniqroq ekanligi ko'rindi.

**MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR**

1. Quyidagi integralning taqribiy qiymatini toping. Bunda $n = 10$ .

2. $\int_0^1 \frac{dx}{1+x}$ $(\approx \ln 2 \approx 0{,}69315)$ integralning taqribiy qiymatini $n = 10$ da to'g'ri to'rtburchaklar formulasi va trapetsiyalar formulasi bo'yicha toping.

3. Trapetsiyalar formulasi bo'yicha $\int_0^1 e^{-x^2} dx$ integralni 0,001 gacha aniqlik bilan hisoblang.

4. $\int_0^3 \frac{dx}{x}$ integralni trapetsiyalar formulasi bo'yicha integrallash oralig'ini 4 bo'lakka bo'lib, hisoblang.

5. $\int_0^1 e^{x^2} dx$ integralni 0,01 gacha aniqlik bilan hisoblang.

6. $\int_0^1 \sqrt{x^2+1} \cdot \sin x dx$ integralni 0,01 gacha aniqlik bilan hisoblang.

7. Trapetsiya formulasi yordamida $n=5$ bo'lganda $\int_0^6 \frac{dx}{\sqrt{8+x^2}}$ integralni taqribiy hisoblang.

8. $n=8$ deb, $\int_2^{10} \frac{dx}{\lg x}$ integralni taqribiy hisoblang.

9. Trapetsiya formulasi bo'yicha $n=8$ da $\int_0^4 \sqrt{2+x^2}dx$ integralni hisoblang.

10. $h=\frac{\pi}{2}$ da $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx$ integralni Simpson formulasi yordamida taqribiy hisoblang.

11. $\int_0^1 \frac{dx}{1+x^2}$ aniq integralni $n=2$ deb olib, Simpson formulasi yordamida hisoblang.

12. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{x} dx$ integralni $2n=10$ da Simpson formulasi yordamida hisoblang.

## XULOSA

Oliy ta'lim muassasalarida Oliy matematikadan o'rganiladigan o'quv materialning har bir mavzusini shu mavzuga tegishli amaliy mazmundagi masalalarni tiplari va turlari bilan bog'lash imkoniyati mavjud va uni amalga oshirish shart va zarur. Matematikani o'qitishda kasbga yo'naltirish samarali shakllardir.

- matematika darslarida amaliy tatbiqiy mazmundagi kasbga yo'naltirilgan masalalar va topshiriqlardan muntazam foydalanish;

- matematika darslarida va kasbiy fanlarni aloqadorlikda o'qitish;

- matematikadan to'garaklarda amaliy mazmunda masalalarni yechishni yo'lga qo'yish va hokozalar.

Oliy ta'lim muassasalarida matematikani o'qitishda interfoal darslarni amalga oshirish bo'yicha ushbu loyihadan quyidagilar kelib chiqadi:

1. O'quvchilarning fanga qiziqishi, olingan ko'nikmalarni kasbga yo'naltirishni amalga oshirish.
2. Mustaqil matematik masalalarni amalga oshirish.
3. Fikrlash, nazariyani amaliyotga tatbiq qilishni bajarish.
4. O'z ustida matematik bilimlarni oshirib borish va xokazalar.

## Mavzu yuzasidan misollar:

Quyidagi integrallarni toping:

1. $\int\left(x^3+2x+\frac{4}{x}\right)dx$;   Javobi: $\frac{x^4}{4}+x^2+4\ln x+c$ .

2. $\int(1-\frac{1}{\cos^2 3x})dx$;   Javobi: $x-\frac{1}{3}tg3x+c$ .

3. $\int\frac{x^2+2x+2}{\sqrt{x^3}}dx$;   Javobi: $\frac{2}{3}\sqrt{x^3}+4\sqrt{x}-\frac{4}{\sqrt{x}}+c$ .

4. $\int 2\sin 3x dx$;   Javobi: $-\frac{2}{3}\cos 3x+c$ .

5. $\int(\frac{1}{3}x^2-6x+\frac{3}{4})dx$;   Javobi: $\frac{x^3}{9}-3x^2+\frac{3}{4}x+c$

6. $\int\sqrt[3]{x^2}dx$;   Javobi: $\frac{3}{5}\sqrt[3]{x^5}+c$.

7. $\int 3\cos 3x dx$.   Javobi: $\sin 3x+c$.

Aniqmas integralda o'zgaruvchini almashtirishga doir misollar:

8. $\int\sqrt{2x}dx;$   Javobi: $\frac{1}{3}\sqrt{(2x)^3}+c$.

9. $\int\sqrt{3x+5}dx;$   Javobi: $\frac{2}{9}\sqrt{(3x+5)^3}+c$.

10. $\int\sin(5x-3)dx;$   Javobi: $-\frac{1}{5}\cos(5x-3)+c$.

11. $\int \cos(16x+5)dx;$ Javobi: $\frac{1}{16}\sin(16x+5)+c.$

12. $\int \frac{xdx}{x^2+1};$ Javobi: $\frac{1}{2}\ln(x^2+1)+c.$

13. $\int \ln(4x-6)dx;$ Javobi: $\frac{4}{4x-6}+c.$

Aniqmas integralni bo‘laklab integrallashga doir misollar:

14. $\int x\sin xdx$; Javobi: $-x\cos x-\sin x+c.$

15. $\int x\cos 2xdx;$ Javobi: $\frac{1}{2}x\sin 2x+\cos 2x+c.$

16. $\int (2x+1)sin3xdx;$ Javobi: $(2x+1)(-\frac{1}{3}cos3x)+\frac{2}{9}sin3x+c.$

17. $\int xarctgxdx;$ Javobi: $\frac{x^2}{2}arc\,tgx-\frac{x}{2}+\frac{1}{2}arctgx+c.$

18. $\int_0^3 x^2dx$; Javobi: 9.

19. $\int_1^2 \frac{1}{x^3}dx$ Javobi: $\frac{3}{8}$.

20. $\int_{-2}^3 2xdx$ Javobi:5.

21. $\int_4^9 \frac{1}{\sqrt{x}}dx$ Javobi: 2.

22. $\int_{-2}^{-1}(5-4x)dx$; Javobi: 11.

23. $\int_{-1}^{1}(x^2+1)dx$; Javobi: $\frac{2}{3}$.

24. $\int_0^{\frac{\pi}{2}} sin(2x+\frac{\pi}{3})dx$ ; Javobi: 0,5.

25. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cos\,xdx$; Javobi: 2.

Quyidagi aniq integrallarni o’zgarivchini almashtirish orqali hisoblang:

26. $\int_0^1 \frac{x^3dx}{(x-1)^2}$, $(x-1=t);$ Javobi: -1,5.

29. $\int_0^5 x\sqrt{x+4}dx,\ (t=\sqrt{x+4})$;Javobi: $33\frac{11}{15}$.

27. $\int_4^9 \frac{dx}{\sqrt{x}-1}$, $(\sqrt{x}=t)$; Javobi: $2(1+\ln 2)$.

28. $\int_3^8 \frac{x}{\sqrt{1+x}}dx$, $(\sqrt{1+x}=t)$; Javobi: $\frac{32}{3}$.

30. $\int_0^1 x\sqrt{1+x^2}dx$, $(t=1+x^2)$; Javobi: $\frac{2\sqrt{2}-1}{3}$.

31. $\int_0^{\frac{\pi}{4}} \frac{dx}{2\cos x+3}$, $(t=tg\frac{x}{2})$; Javobi: $\frac{2}{\sqrt{5}}arctg\frac{1}{\sqrt{5}}$.

Quyidagi aniq integrallarni bo'laklab integrallash orqali hisoblang:

32. $\int_0^e xe^x dx$, $(u=x)$; Javobi: $e^e(e-1)+1$.

33. $\int_0^{\pi} x^2 \ln x dx$, $(u=\ln x)$; Javobi: 2.

34. $\int_0^{\frac{\pi}{2}} x\cos x dx$, $(u=x)$; Javobi: -2.

35. $\int_0^1 arctgx dx$, $(u=arctgx)$; Javobi: $\frac{\pi}{4}-\ln\sqrt{2}$.

36. $\int_0^{\pi} x\cos x dx$, $(u=x)$; Javobi: 2.

37. $\int_1^8 x\sin x dx$, $(u=x)$; Javobi: $\pi$.

## Nazorat savollari:

1.Integrallar jadvalidagi o‘zingiz xohlagan 4 ta formulani tanlang va uni isbotlang.

2. Integrallashning sodda qoidalarini bayon qiling. Misollarda tushuntiring.

3. O‘zgaruvchi almashtirish usuli nima?

4.Aniq integral nima?

5. Egri chiziqli trapetsiya yuzini hisoblash masalasini ayting. Misollarda tushuntiring.

6. Nyuton–Leybnis formulasi nima? Uning mazmun-mohiyatini ayting.

7. Aniq integralning xossalarini ayting. Misollarda tushuntiring.

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